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DIFFERENTIAL INCIDENCE IN A STATIC GENERAL EQUILIBRIUM FRAMEWORK*

by

Adolph Vandendorpe
Ann F. Friedlaender

Number 112B June 1973

massachusetts
institute of
technology
50 memorial drive
cambridge, mass. 02139

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[*] The research for this paper was done while Vandendorpe and Friedlaender were respectively a visiting shcolar and visiting professor at M.I.T. Discussions with James Anderson were helpful in developing this paper. In addition to providing helpful comments on an earlier draft of this paper, Peter Diamond has provided substantial insights into the nature of fiscal theory.

1. Introduction

The analysis of differential tax incidence is concerned with the impact on the variables of a specified economic system of a change in the tax structure that permits the government to maintain its same real demand. Since real government demand is assumed fixed, any change in the equilibrium of the system can be attributed to the change in the tax structure alone. Thus the analysis of differential incidence abstracts from any government expenditure effects.[1]

Within this context, the analysis of Harberger (1962) and Mieszkowski (1967), carried out within the standard two-sector, two-factor model (with fixed endowments of both factors), has become the standard static general equilibrium analysis of differential incidence.[2] Harberger and Mieszkowski (hereafter referred to as HM) examine the impact on relative factor prices of commodity taxes and sectoral factor taxes under the following simplifying assumptions:[3]

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[1] For a discussion of other variants of incidence analysis see, for example, Musgrave (1953, 1959) and Mieszkowski (1969). For a discussion of the importance of maintaining full employment see McLure (1970) and Friedlaender and Due (1973).

[2] For an interesting analysis of dynamic incidence see Feldstein (1972).

[3] Strictly speaking, incidence analysis is ultimately concerned with the impact upon welfare of a change in the tax structure. Since, however, commodity prices, output, consumption, and thus welfare ultimately depend on relative factor prices in this model, the analysis is limited to the impact of a change in the tax structure on relative factor prices alone.

(i) Initial taxes on factors and commodities are zero.

(ii) Real government demand is zero.

(iii) The government spends the proceeds of taxation in the same manner as consumers.[4]

Assumption (iii), in conjunction with assumption (ii), has the equivalent interpretation, more akin to the concept of differential incidence discussed above, that all revenue raised by commodity or factor taxes is returned as a lump sum subsidy to consumers. Assumption (i), however, has the important implication that, as long as the analysis is carried out in terms of marginal changes and first-order comparative statics,[5] there is no "deadweight loss" associated with distorting changes in the tax structure. Thus if the government returns to consumers as a lump sum payment any revenues raised by distorting commodity and factor taxes, the consumer is kept (locally) on the same indifference curve. Consequently, there is no need to study explicitly the revenue effects of taxation on the government budget constraint nor the income effects on consumer demand; all of this can effectively be dealt with by assuming a joint (government plus private) aggregate demand function of which only the substitution effect is relevant to the analysis.

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[4] A single aggregate private demand function is assumed in Harberger (1962) and in the first part of Mieszkowski (1967). Mieszkowski then considers the case where capitalists and workers have different demand functions. In the present paper, we shall not address ourselves to the latter case.

[5] A more appropriate label to the HM analysis as well as that of the present paper would therefore be "marginal differential incidence."

Thus these simplifying assumptions eliminate many of the interesting general equilibrium implications of differential incidence analysis. In fact, the results derived from the HM general equilibrium framework are qualitatively identical in content to those of the standard partial equilibrium analysis of tax incidence. In the usual partial-equilibrium, supply-demand analysis, the effect on the producer price (p) of imposing an ad valorem tax can be written as

$$\frac{\hat{p}}{\hat{T}} = \frac{E_d}{E_s - E_d} \qquad (1)$$

where T represents the tax coefficient (i.e., one plus the ad valorem tax rate), E_d and E_s respectively represent the price elasticity of demand and supply, and the notation $\hat{x}$ denotes dx/x.[6] In the HM framework, the effect on the producer commodity price ratio (p_1/p_2) of introducing an ad valorem tax on good 1 (when there are no other existing factor or commodity taxes) can be written as[7]

$$\frac{\hat{p}_1 - \hat{p}_2}{\hat{T}_1} = \frac{-\sigma_D}{\sigma_S + \sigma_D} \qquad (2)$$

where σ_D and σ_S are respectively the elasticities of substitution in demand and supply. Since σ_D has been defined to be positive, eqs. (1) and (2) are readily seen to be qualitatively similar.[8]

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[6] Differentiation of the supply-demand equations, $S(p) = D(pT)$, yields $\hat{p}E_s = E_d(\hat{p} + \hat{T})$. Solving for $\hat{p}/\hat{T}$ (the percentage change in p corresponding to a one percent change in the tax coefficient T) yields eq. (1).

[7] Equation (2) will be derived below.

[8] Jones (1965, eqs. (13)-(14)) derives an analogous formula for the effect of producer subsidies on relative prices in his two-sector model with no explicit government demand, but an aggregate homothetic utility function.

Similarly, the effect on relative producer prices (net of factor taxes) of introducing a capital tax in sector 1 is given in the HM framework as[9]

$$\frac{\hat{\pi}_1 - \hat{\pi}_2}{\hat{T}_{K1}} = \frac{-(\theta_{K1}\sigma_D + s_1)}{\sigma_S + \sigma_D} \qquad (3)$$

where π_j represents the producer price of commodity j net of factor taxes; T_{K1} represents one plus the ad valorem rate of the capital tax in sector 1; s_1 represents the (substitution) elasticity of the output ratio X_1/X_2 with respect to the factor cost ratio in sector 1; and θ_{K1} represents the relative share of capital in sector 1. Again, equation (3) is completely analogous to a partial equilibrium adjustment in the market price (ratio) resulting from an exogenous shift in parameters affecting both the supply and demand curves.[10]

The above paragraphs clearly indicate that the simplifying assumptions of the HM analysis are indeed costly in terms of the general-equlibrium quality of the results obtained. Relaxing the assumptions (specifically assumption (i)) is thus likely to permit an analysis which is of a more genuine general-equilibrium nature (even under the assumption of an aggregate private-demand function) in that the crucial elements of deadweight loss effects, revenue effects on the government budget, and income effects on consumer demand will be brought to the foreground.[11]

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[9] This is equation (12) in Harberger (1962), which is repeated as equation (1) in Mieszkowski (1967). The precise translation of HM's notation into our notation and the derivation of equation (3) will be given below.

[10] A $\hat{T}_{K1}$ percent increase in the factor tax coefficient T_{K1} (at constant factor rentals) affects the ratios of quantitites supplied, say X_1/X_2, by $s_1\hat{T}_{K1}$ percent (a quantity that may be positive or negative, depending upon the relative factor intensities in both sectors); it increases the consumer price ratio by $\theta_{K1}\hat{T}_{K1}$ percent, and hence reduces the ratio of quantitites demanded, say C_1/C_2, by $\theta_{K1}\sigma_D\hat{T}_{K1}$ percent.

Briefly this paper takes the following form. The next section provides an outline of the model and gives an overview of the nature of the analysis of differential incidence. Section 3 then derives the differentials of the production and consumption relationships, which are used to obtain the reduced-form equations of the government budget constraint and the market clearing condition, whose derivation is given in Section 4. Section 5 then discusses the nature of the deadweight losss effects while section 6 analyzes the nature of the equilibrating adjustments in response to changes in the tax structure. Section 7 then provides a brief summary of the findings of this paper.

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[11] To put things in perspective, it should be pointed out, however, that a two-sector analysis will be inherently somewhat disappointing to the reader looking for multi-market interaction since Walras' law permits us to concentrate on a single market clearing equation.

2. The Model, Notation and Overview of the Analysis

Following Jones' (1965, 1971) treatment of the two-commodity, two-factor model, the basic equilibrium relationships consist of the full employment equations,

$$a_{K1}X_1 + a_{K2}X_2 = \bar{K} \ , \tag{4a}$$

$$a_{L1}X_1 + a_{L2}X_2 = \bar{L} \ , \tag{4b}$$

and the competitive zero-profit conditions,

$$a_{K1}r_1 + a_{L1}w_1 = p_1 \ , \tag{5a}$$

$$a_{K2}r_2 + a_{L2}w_2 = p_2 \ , \tag{5b}$$

where $\bar{K}$ and $\bar{L}$ are the inelastically supplied fixed endowments of capital and labor; X_j is the output of commodity j; r_j and w_j represent the cost (inclusive of factor taxes) of employing respectively one unit of capital or labor in sector j; p_j is the producer-price of commodity j; a_{ij} is the amount of factor i used per unit of commodity j.[12],[13] Because of the possibility of distorting factor taxes r_1 (or w_1) need not be equal to r_2 (or w_2). Indeed, if r and w represent the rental and wage rates respectively received by capital and labor (in both sectors) then we can define the tax related symbols t_{ij}, τ_{ij} and T_{ij} by

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[12] In the above paragrph as well as in the rest of the paper the subscript j refers to commodities (sectors) 1 and 2, while the subscript i refers to factors K and L. Unless clarity demands it, this range of the subscripts i and j will not be repeated every time.

[13] It should be noted that a_{ij} is a function of the factor cost ratio r_j/w_j, defined by the unit isoquants and the cost-minimizing tangency conditions.

$$r_j \equiv r + t_{Kj} = r(1 + \tau_{Kj}) = rT_{Kj} \tag{6a}$$

$$w_j \equiv w + t_{Lj} = w(1 + \tau_{Lj}) = wT_{Lj} \tag{6b}$$

Thus t_{ij} represents the specific tax and τ_{ij} the ad valorem tax rate on the use of factor i in sector j. The term, "tax coefficient", will be used for the symbol T_{ij} $(\equiv 1 + \tau_{ij})$.

On the (private-) demand side of the model we define C_j as the quantity of good j privately consumed, and we postulate aggregate private demand functions,[14] given by

$$C_j = C_j(q_1, q_2, y) , \tag{7}$$

where q_j represents the price facing consumers and y represents aggregate disposable money income. The relationship between consumer prices (q_j) and producer prices (p_j) is given by the following relationships

$$q_j \equiv p_j + t_j \equiv p_j(1 + \tau_j) = p_jT_j \tag{8}$$

where t_j represents the specific tax and τ_j represents the ad valorem tax rate on commodity j. Again we refer to T_j $(\equiv 1 + \tau_j)$ as the tax coefficient on commodity j.

Disposable income is given by factor income less lump sum taxes

$$\begin{aligned} y &= M - H \\ &= r\bar{K} + w\bar{L} - H \end{aligned} \tag{9}$$

where H represents lump sum taxes collected by the government.

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[14] For simplicity, we can think of this as a one consumer economy or its equivalent. See Samuelsom (1956).

The demand equations (7) are assumed to satisfy the aggregate consumer budget constraint. Thus for all q_j and y

$$\sum_j q_j C_j(q_1, q_2, y) = y \tag{10}$$

The government is assumed to purchase fixed quantities, G_j, of the two final goods at producer prices[15] and to have at its disposal seven tax instruments: the neutral lump sum tax (H), two commodity tax coefficients (T_j), and four sectoral factor tax coefficients (T_{ij}).[16] Taking note of the tax relationships given in eqs. (6) and (8), we write the government budget constraint as[17]

$$\sum_j t_j C_j + \sum_{i,j} t_{ij} v_{ij} + H = \sum_j p_j G_j \tag{11}$$

where v_{ij} denotes the amount of factor i employed in sector j.

Finally, we can write the market clearing equations as

$$C_j + \bar{G}_j = X_j \tag{12}$$

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[15] Equivalently, we could assume that the government imposed commodity taxes on total output X_j and purchased its demand at consumer prices q_j.

[16] It is a matter of choice whether specific tax rates, ad valorem tax rates, or tax coefficients serve as the ultimate policy instruments since eqs. (6) and (8) provide the definitional links between them. In the present formulation of the model it is generally notationally most convenient to deal in tax coefficients, although this procedure necessitates some care in correctly interpreting our results. We shall return to this point when confusion possible.

[17] When dealing with the government budget constraint it is usually simplest to use specific tax rates. Note that these are related to the tax coefficients and price variables: $t_j \equiv (T_j - 1)p_j$; $t_{Kj} \equiv (T_{Kj} - 1)r$; and $t_{Lj} \equiv (T_{Lj} - 1)w$.

Equations (4)-(12) specify the model and permit us to determine the equilibrium output, consumption, factor allocation, prices, and factor returns in terms of a given tax structure. To analyze marginal differential incidence, we can conceptually specify an exogenous change in the tax structure and conceptually totally differentiate the system and solve for the equilibrating changes in the relevant variables.

Equations (11) and (12), however, form the basis of the analysis. Since Walras' law permits us to eliminate one of the market clearing equations, our first goal is to derive the reduced form of the differentials of the government budget constraint and one of the market clearing equations.[18] Thus we ultimately describe the differential of the government budget constraint and one of the market clearing equations in terms of eight differential elements: the percentage change in relative factor prices $(\hat{r} - \hat{w})$; real lump sum taxes $(\hat{H} - \hat{w})$; commodity tax coefficients $(\hat{T}_j)$; and factor tax coefficients $(\hat{T}_{ij})$. Since the differential of the market clearing equation permits us to eliminate $(\hat{r} - \hat{w})$, we can then write the differential of the government budget constraint in terms of seven tax differentials alone: $\hat{T}_j$, $\hat{T}_{ij}$, and $(\hat{H} - \hat{w})$.[19]

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[18] Actually, in the ensuing analysis we use a linear combination of the differentials of the two market clearing equations and hence effectively reduce them to one equation.

[19] The term $(\hat{H} - \hat{w})$ denotes change in real lump sum taxes. The use of this expression implies that only the change in the real lump sum tax can be determined in this analysis, instead of the money change. This, of course, is a natural outgrowth from the fact that the supply-demand analysis of this model can only determine relative prices. We can alternatively view the rate of inflation in the general price level (or the absolute price level) as being controlled by the government. The fact that H is divided by w is not intended to give labor an asymmetric treatment as a numéraire. However, w emerges as the most convenient index of the change in absolute prices, although any other "general price index" would have served the same purpose. Since the ensuing analysis indicates that $\hat{H}$ and $\hat{w}$ are tied together, we treat them as one variable, the percentage change in real lump sum taxes, rather than as two separate variables, the percentage change in money lump sum taxes and the percentage change in wages (as an indicator of the absolute price level).

$$\sum_{j} m_j \hat{T}_j + \sum_{i,j} m_{ij} \hat{T}_{ij} + m_h (\hat{H} - \hat{w}) = 0 \qquad (13)$$

The determination of the coefficients m_j, m_{ij}, and m_h is one of the main objects of the subsequent analysis. The appropriate ratio of these coefficients (with a minus sign) represents the marginal rate at which the government can substitute one tax for another and still be assured that it can exactly pay (at different prices) for the same bundle of real goods $(\bar{G}_1, \bar{G}_2)$ after all general equilibrium repercussions have worked themselves out.[20] Equation (13) may therefore be called the general equation of differential incidence. From eq. (13) we can readily see that the government has six degrees of freedom in the marginal change in the tax structure. The effect of any of these admissible changes in the tax structure upon the change in relative factor prices $(\hat{r} - \hat{w})$ can be determined by substituting equation (13) into the reduced form of the differential of the market clearing constraint. Although the analysis of differential incidence usually stops with determining the effect of changes in the tax structure upon $(\hat{r} - \hat{w})$, once we have determined $(\hat{r} - \hat{w})$ we could then determine the change in relative prices and hence in output and consumption.

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[20] Strictly speaking these ratios do not represent "marginal rates of substitution", since the tax coefficients are expressed in percentage terms.

3. The Equations of Change

To determine the reduced form of the market clearing equation and government budget constraint, we must differentiate the basic production and consumption relationships given above. These relationships and their associated equations of change have been elegantly synthesized by Jones (1965, 1971). In this section we shall therefore provide a brief summary of the basic equations of change in consumption and production, adapted to the present analysis. As much as possible, we shall adhere to Jones' symbols and notation.

3.1 The Production Relationships

We begin by differentiating equations (4) and (5) and obtain, taking into account that $\hat{K} = \hat{L} = 0$,

$$\lambda_{K1}\hat{X}_1 + \lambda_{K2}\hat{X}_2 = -(\lambda_{K1}\hat{a}_{K1} + \lambda_{K2}\hat{a}_{K2}) \; , \tag{14a}$$

$$\lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2 = -(\lambda_{L1}\hat{a}_{L1} + \lambda_{L2}\hat{a}_{L2}) \; , \tag{14b}$$

$$\theta_{K1}\hat{r}_1 + \theta_{L1}\hat{w}_1 = \hat{p}_1 - (\theta_{K1}\hat{a}_{K1} + \theta_{L1}\hat{a}_{L1}) \; , \tag{15a}$$

$$\theta_{K2}\hat{r}_2 + \theta_{L2}\hat{w}_2 = \hat{p}_2 - (\theta_{K2}\hat{a}_{K2} + \theta_{L2}\hat{a}_{L2}) \; , \tag{15b}$$

where λ_{ij} represents the fraction of the total endowment of factor i allocated to sector j (i.e., $\lambda_{Kj} = \frac{a_{Kj}X_j}{\bar{K}}$ and θ_{ij} represents the share of the costs of factor i (including taxes) in the unit production costs of commodity j (i.e., $\theta_{Kj} = r_j a_{Kj}/p_j$).[21]

The elasticity of factor substitution in sector j can be written as

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[21] Note that due to the full employment condition, $\lambda_{i1} + \lambda_{i2} = 1$, and that due to the zero profit condition, $\theta_{Kj} + \theta_{Lj} = 1$.

$$\sigma_j = -\frac{\hat{a}_{Kj} - \hat{a}_{Lj}}{\hat{r}_j - \hat{w}_j}, \tag{16}$$

and the first order conditions of cost minimizations imply

$$\theta_{Kj}\hat{a}_{Kj} + \theta_{Lj}\hat{a}_{Lj} = 0 . \tag{17}$$

Hence from equations (16) and (17) each $\hat{a}_{ij}$ can be solved for in terms of $(\hat{r}_j - \hat{w}_j)$:

$$\hat{a}_{Kj} = -\theta_{Lj}\sigma_j(\hat{r}_j - \hat{w}_j) \tag{18a}$$

$$\hat{a}_{Lj} = \theta_{Kj}\sigma_j(\hat{r}_j - \hat{w}_j) \tag{18b}$$

Substitution of equations (18) into equations (14) gives

$$\lambda_{K1}\hat{X}_1 + \lambda_{K2}\hat{X}_2 = \delta_{K1}(\hat{r}_1 - \hat{w}_1) + \delta_{K2}(\hat{r}_2 - \hat{w}_2) \tag{19a}$$

$$\lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2 = -\delta_{L1}(\hat{r}_1 - \hat{w}_1) - \delta_{L2}(\hat{r}_2 - \hat{w}_2) \tag{19b}$$

where $\delta_{Kj} = \lambda_{Kj}\theta_{Lj}\sigma_j$ and $\delta_{Lj} = \lambda_{Lj}\theta_{Kj}\sigma_j$. Thus δ_{Kj} represents the rate (as a percentage of the total capital stock $\bar{K}$) at which capital is released by sector j as a result of the increase in its relative cost, and δ_{Lj} represents the rate (as a percentage of the total labor endowment $\bar{L}$) at which labor is absorbed in sector j as a result of the decline in its relative cost.

Solving equations (19) for $\hat{X}_1$ and $\hat{X}_2$ and taking into account the factor tax relationships given in equations (6), we obtain

$$\hat{X}_1 = (S_{11} + S_{12})(\hat{r} - \hat{w}) + S_{11}(\hat{T}_{K1} - \hat{T}_{L1}) + S_{12}(\hat{T}_{K2} - \hat{T}_{L2}) \tag{20a}$$

$$\hat{X}_2 = (S_{21} + S_{22})(\hat{r} - \hat{w}) + S_{21}(\hat{T}_{K1} - \hat{T}_{L1}) + S_{22}(\hat{T}_{K2} - \hat{T}_{L2}) \tag{20b}$$

where

$$S_{1j} = \frac{1}{|\lambda|}(\lambda_{L2}\delta_{Kj} + \lambda_{K2}\delta_{Lj}) \qquad j = 1,2$$

$$S_{2j} = \frac{-1}{|\lambda|}(\lambda_{L1}\delta_{Kj} + \lambda_{K1}\delta_{Lj}) \qquad j = 1,2$$

$$|\lambda| = \lambda_{K1} - \lambda_{L1} = \lambda_{L2} - \lambda_{K2}.$$ [22]

Thus S_{fj} (f = 1,2; j = 1,2) represents the elasticity of supply of good f with respect to the factor cost ratio (r_j/w_j) in sector j. Therefore $S_f \equiv (S_{f1} + S_{f2})$ represents the elasticity of good f with respect to changes in the factor cost ratio.

3.2 The Producer Price Relationships

Equations (20) are the basic general equilibrium supply relationships describing the effects of changes in relative factor prices and factor tax coefficients on output. We also need the relationships between the various price changes and changes in the tax coefficients. To obtain these, we differentiate the zero profit conditions (5), and, taking into account (17) and (6), we obtain

$$\theta_{K1}\hat{r} + \theta_{L1}\hat{w} = \hat{p}_1 - (\theta_{K1}\hat{T}_{K1} + \theta_{L1}\hat{T}_{L1}) \tag{21a}$$

$$\theta_{K2}\hat{r} + \theta_{L2}\hat{w} = \hat{p}_2 - (\theta_{K2}\hat{T}_{K2} + \theta_{L2}\hat{T}_{L2}) \tag{21b}$$

It is useful to define the symbols

$$\hat{T}_{fj} \equiv \theta_{Kj}\hat{T}_{Kj} + \theta_{Lj}\hat{T}_{Lj} \tag{22}$$

and

$$\hat{\pi}_j \equiv \theta_{Kj}\hat{r} + \theta_{Lj}\hat{w} . \tag{23}$$

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[22] In computing the value of the determinant of $\lambda = [\lambda_{ij}]$, use is made of $\lambda_{i1} + \lambda_{i2} = 1$.

$\hat{T}_{fj}$ represents the percentage change in the producers' price of X_j due to changes in factor tax coefficients, while $\hat{\pi}_j$ represents the percentage change in the producers' price of X_j due to changes in factor prices. Taking the commodity tax relationships (8) into account, we can succinctly summarize the relationships between price changes and changes in the tax structure by

$$\hat{q}_j = \hat{p}_j + \hat{T}_j = \hat{\pi}_j + \hat{T}_{fj} + \hat{T}_j \ . \qquad (24)$$

Equation (24) breaks down the (percentage) change in consumer prices (q_j) into three components: $\hat{\pi}_j$, the change in factor rental cost (a share-weighted average of the change in factor prices); $\hat{T}_{fj}$, the change in factor tax costs (a share-weighted average of the change in factor tax coefficients); and $\hat{T}_j$, the change in the commodity tax coefficient.

For future reference, we also note that eq. (23) implies

$$\hat{\pi}_1 - \hat{\pi}_2 = |\theta|(\hat{r} - \hat{w}) \qquad (25)$$

where $|\theta| = \theta_{K1} - \theta_{K2} = \theta_{L2} - \theta_{L1}$.[23]

3.3 The Consumer Demand Relationships

Finally, we must express the changes in consumer prices in terms of changes in taxes and factor prices. Differentiation of the demand equation (7) yields the standard compensated demand relationships

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[23] In computing the determinant of $\theta \equiv [\theta_{ij}]$ we make use of the fact that $\theta_{Kj} + \theta_{Lj} = 1$.

$$\hat{C}_1 = E_{11}(\hat{q}_1 - \hat{q}_2) + \eta_1(\hat{y} - \gamma_1\hat{q}_1 - \gamma_2\hat{q}_2) \qquad (26a)$$

$$\hat{C}_2 = - E_{22}(\hat{q}_1 - \hat{q}_2) + \eta_2(\hat{y} - \gamma_1\hat{q}_1 - \gamma_2\hat{q}_2) \qquad (26b)$$

where E_{11} is the compensated elasticity of C_1 with respect to q_1/q_2; E_{22} is the compensated elasticity of C_2 with respect to q_2/q_1; η_j is the (money) income elasticity of good j; and γ_j is the fraction of disposable income spent on good j (i.e. $\gamma_j = q_jC_j/y$).

By differentiating eq. (9) we obtain the change in disposable income

$$\hat{y} = \varphi_K\hat{r} + \varphi_L\hat{w} + h\hat{H} \qquad (27)$$

where $\varphi_K = r\bar{K}/y$, $\varphi_L = wL/y$ and $h = H/y$

By substituting eqs. (23-24) and (27) into eqs. (26) and gathering terms we then obtain

$$\hat{C}_j = (\hat{r} - \hat{w})[E_{jj} + \eta_j\varphi] + E_{11}[(\hat{T}_1 - \hat{T}_2) + \hat{T}_{\ell 1} - \hat{T}_{\ell 2})] \qquad (28)$$
$$- \eta_j[\sum_j \gamma_j(\hat{T}_j + \hat{T}_{\ell j}) + h(\hat{H} - \hat{w})]$$

where $\varphi = \varphi_K - \sum_j \gamma_j\theta_{Kj}$.

4. The Basic Reduced Form Equations

In this section we substitute the equations of change developed in the previous section into the differentials of the market clearing condition (12) and the government budget constraint (11). In doing so, we develop a system of two equations in the eight differential variables: $(\hat{r} - \hat{w})$, two $\hat{T}_j$'s, four $\hat{T}_{ij}$'s, and $(\hat{H} - \hat{w})$.

Differentiation of the market clearing equation (12) yields

$$\hat{X}_j = g_j \hat{C}_j \tag{29}$$

where $g_j = C_j/X_j$.

To capture the supply and demand responses in both markets, we use a linear combination of both market clearing equations instead of one of them alone. Thus equation (29) permits us to write

$$\hat{X}_1 - \hat{X}_2 = g_1 \hat{C}_1 - g_2 \hat{C}_2 \ . \tag{30}$$

Let us now introduce the following symbols:

$$\tilde{\eta} \equiv g_1 \eta_1 - g_2 \eta_2$$

$$\tilde{\sigma}_D \equiv -(g_1 E_{11} + g_2 E_{22})$$

$$\sigma_S \equiv \frac{S_1 - S_2}{|\theta|}$$

$$s_1 \equiv S_{11} - S_{21}$$

$$s_2 \equiv S_{12} - S_{22} \ .$$

Using equations (20) and (28), equation (30) can be rewritten as

$$(\hat{r}-\hat{w})[\,|\theta|(\sigma_S+\tilde{\sigma}_D)-\tilde{\eta}\varphi]+[(\hat{T}_1-\hat{T}_2)+(\hat{T}_{\ell 1}-\hat{T}_{\ell 2})]\tilde{\sigma}_D$$

$$+\tilde{\eta}\sum_j(\hat{T}_j+\hat{T}_{\ell j})\gamma_j+\sum_j(\hat{T}_{Kj}-\hat{T}_{Lj})s_j+\tilde{\eta}h(\hat{H}-\hat{w})=0 \qquad (31)$$

We now turn to the government budget constraint (11), which we differentiate totally to obtain[24]

$$\sum_j t_j dC_j + \sum_{i,j} t_{ij} dv_{ij} + \sum_j C_j dt_j + \sum_{i,j} v_{ij} dt_{ij} + dH = \sum_j \bar{G}_j dP_j \qquad (32)$$

It is useful to divide eq. (32) into two parts. The first two terms measure the change in revenue due to the existing tax structure, while the remaining terms measure the net change in revenue arising directly from the change in the tax structure and the associated price effects. We can simplify eq. (32) by making use of the following three relationships.

$$\sum_j C_j dt_j = \sum_j C_j dq_j - \sum_j X_j dp_j + \sum_j \bar{G}_j dp_j \qquad (33)$$

$$\sum_{i,j} v_{ij} dt_{ij} = \sum_j X_j dp_j - (\bar{K}dr + \bar{L}dw) \qquad (34)$$

$$\sum_{i,j} t_{ij} dv_{ij} = \sum_j p_j dX_j \qquad (35)$$

Equation (33) follows directly from the market clearing equations (12) and the tax defintions (8). Equation (34) and (35) are important implications of the production side of the model.[25]

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[24] Again, we note that the government budget constraint is currently expressed in terms of specific taxes, which will shortly be translated into equivalent tax coefficients.

[25] By taking eq. (17) into account, we may rewrite eqs. (15) as,

$$a_{K1}dr_1 + a_{L1}dw_1 = dp_1 \qquad a_{K2}dr_2 + a_{L2}dw_2 = dp_2$$

Multiplying the first of these equations by X_1 and the second by X_2 and adding gives

$$\sum_j v_{Kj} dr_j + \sum_j v_{Lj} dw_j = \sum_j X_j dp_j \,.$$

(footnote continued on p. 18)

Substituting equations (32-35) into eq. (32) and dividing by y we obtain

$$\sum_j \rho_j \hat{X}_j + \sum_j \zeta_j \hat{C}_j + \sum_j \gamma_j q_j - \hat{y} = 0 \tag{36}$$

where $\rho_j = p_j X_j / y$ and $\zeta_j = t_j C_j / y$.[26]

The last two terms of this expression reflect the direct effects of a change in the tax structure and its associated price effect. Using eqs. (23), (24), and (27), we may rewrite these two terms as

$$\sum_j \gamma_j \hat{q}_j - \hat{y} = -\varphi(\hat{r} - \hat{w}) + \gamma_1(\hat{T}_{f1} + \hat{T}_1) + \gamma_2(\hat{T}_{f2} + \gamma_2) + h(\hat{H} - \hat{w}). \tag{37}$$

The first two terms of eq. (36) reflect the revenue effects of a change in the tax structure due to the existing tax structure. Thus the revenue effect of existing commodity taxes is given by $\sum_j \zeta_j \hat{C}_j$ while the revenue effect of existing factor taxes is given by $\sum_j \rho_j \hat{X}_j$.

Using eqs. (24) and (28) we may write

$$\sum_j \zeta_j \hat{C}_j = E[\,|\theta|(\hat{r} - \hat{w}) + (\hat{T}_1 - \hat{T}_2) + (\hat{T}_{f1} - \hat{T}_{f2})]$$

$$- \sum_j \zeta_j \eta_j \Big(\sum_j \gamma_j \hat{q}_j - \hat{y} \Big) \tag{38}$$

where $E = \zeta_1 E_{11} - \zeta_2 E_{22}$.

(footnote continued from p. 17)
By writing $dr_j = dr + dt_{Kj}$ and $dw_j = dw + dt_{Lj}$ and making use of the full employment conditions (4), we can rewrite this last expression as

$$Kdr + Ldw + \sum_{i,j} v_{ij} dt_{ij} = \sum_j X_j dp_j,$$ which is eq. (34)

The constant returns to scale assumption also permits us to write

$$rK + wL + \sum_{i,j} t_{ij} v_{ij} = p_j X_j \; .$$

Total differentiation of the above yields

$$Kdr + Ldw + \sum_{i,j} v_{ij} dt_{ij} + \sum_{i,j} t_{ij} dv_{ij} = \sum_j p_j dX_j + \sum_j X_j dp_j \; .$$

Subtracting eq. (34) from this last equation yields equation (35).

[26] Since $t_j = \Upsilon_j p_j$, we may readily express ζ_j in terms of ad valorem taxes and write $\zeta_j = \Upsilon_j p_j C_j / y$.

The first term of eq. (38) represents the change in government revenue (as a percentage of disposable income) arising from the substitution effects in consumption due to a one percent change in the consumer price ratio q_1/q_2. While this will obviously equal zero if there are no initial taxes, it will also equal zero, if the initial tax structure is nondistorting.[27] The second term of eq. (38) measures the change in revenue due to the income effect of consumption. This term will generally only equal zero if initial taxes are zero.[28]

Using eq. (20) we obtain

$$\sum_j \rho_j \hat{X}_j = \epsilon(\hat{r}-\hat{w}) + \epsilon_1(\hat{T}_{K1} - \hat{T}_{L1}) + \epsilon_2(\hat{T}_{K2} - \hat{T}_{L2}) \tag{39}$$

where $\epsilon_j = \rho_1 S_{1j} + \rho_2 S_{2j}$ and $\epsilon \equiv \epsilon_1 + \epsilon_2$. By expanding $S_{\ell j}$ we can see that ϵ_j represents the change in government revenue (as a percentage of disposable income) due to the substitution effects in factor usage associated with a one percent change in the factor cost ratio in industry j. These substitution effects will also

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[27] Using the fact that $\gamma_1 E_{11} = \gamma_2 E_{22}$ we can readily see that

$$E = \frac{E_{11} p_1 C_1}{1 + \tau_1} (\tau_1 - \tau_2).$$

Thus if the initial taxes are nondistorting, $\tau_1 = \tau_2$ and $E = 0$.

[28] As long as both goods are superior so that $\eta_j > 0$, then $\sum_j \zeta_j \eta_j = 0$ if $\zeta_j = 0$. If, however, one $\eta_j < 0$, it is possible for $\sum_j \zeta_j \eta_j = 0$ even if $\zeta_j \neq 0$.

equal zero if initial taxes are zero or nondistorting.[29]

Thus equations (38) and (39) indicate that the change in revenues arising from the initial tax structure is due to an income effect in consumption and the substitution effects in consumption and production. In the absence of initial taxes, all of these effects will be obviously zero. Less obviously, if initial taxes are nondistorting, these substitution effects are also zero and the income effect alone affects revenues.

Substituting eqs. (37)-(39) into eq. (36) and collecting terms, we finally obtain

$$(\hat{r}-\hat{w})[\epsilon + |\theta|E - m\varphi] + [(\hat{T}_1 - \hat{T}_2) + (\hat{T}_{\ell 1} - \hat{T}_{\ell 2})]E$$

$$+\sum_j (\hat{T}_{Kj} - \hat{T}_{Lj})\epsilon_j + m[h(\hat{H} - \hat{w}) + \sum_j \gamma_j(\hat{T}_j + \hat{T}_{\ell j})] = 0 \qquad (40)$$

where $m = 1 - \sum_j \zeta_j \eta_j$.

Equation (40) thus represents the differential of the government budget constraint expressed in terms of $(\hat{r}-\hat{w})$, $\hat{T}_j$, $\hat{T}_{ij}$ and $(\hat{H} - \hat{w})$. The first term represents the change in relative factor prices caused by the change in the tax structure. The second two terms

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[29] From eq. (20) we expand $S_{\ell j}$ and derive

$$\epsilon_1 = \rho_1 S_{11} + \rho_2 S_{21} = \frac{\lambda_{L1}\lambda_{K1}}{|\lambda|}(\sigma_1)(\rho_2)[\varphi_{L2}(T_{L1} - T_{L2}) + \varphi_{K2}(T_{K1} - T_{K2})]$$

$$\epsilon_2 = \rho_1 S_{12} + \rho_2 S_{22} = \left(\frac{\lambda_{K2}\lambda_{L2}}{|\lambda|}\right)(\sigma_2)(\rho_1)[\varphi_{L1}(T_{L1} - T_{L2}) + \varphi_{K1}(T_{K1} - T_{K2})].$$

Nondistorting initial taxes imply $T_{K1} = T_{K2} = T_{L1} = T_{L2}$. Hence if initial taxes are zero or nondistorting, $\epsilon_1 = \epsilon_2 = \epsilon = 0$.

represent the changes in revenues arising from the substitution effects in consumption and production due to the existing tax structure. The final term represents the change in revenue arising directly from a change in the tax structure. The factor m reflects the sum of two effects. A rise in factor and commodity tax coefficients or in real lump sum taxes produces an immediate increase in revenues. But it also reduces real income, which in turn reduces consumption through the income effect. This in turn has a negative effect on revenue through the existing tax structure. Hence the factor m $(=1-\sum_j \zeta_j\eta_j)$ represents the net effect on revenues of a change in taxes.

5. Deadweight Loss Effects

The analysis of the government budget constraint also throws light on the concept of (first-order) deadweight loss in a general equilibrium system. Deadweight loss arises whenever the collection of revenue through distorting taxes involves a welfare cost to consumers beyond the money collected; that is, if the government were to return in lump sum fashion the money collected through distorting taxes, the consumers would still be worse off than they were in the absence of the distorting taxes.

In the usual first-order comparative static analysis, however, the deadweight loss associated with introducing new distorting taxes, analytically only shows up in the presence of distoring initial taxes. Consequently, whenever initial tax rates are assumed to be equal or zero, an important effect of distoring taxation is automatically assumed away.

The best way to see this in the context of the present model is to solve the differential of government budget equation (32) for dH in the following way

$$dH = -[\sum_j C_j dq_j - (\bar{K}dr + \bar{L}dw)]$$

$$-\mu[(E_{11} t_1 C_2 - E_{22} t_2 C_2)(\hat{q}_1 - \hat{q}_2) + \sum_{i,j} t_{ij} dv_{ij}] \qquad (41)$$

where $\mu = 1/m$.

Equation (41) indicates that we may separate the change in the lump sum tax that will equilibrate the government's budget constraint into two parts. The first part is readily identified through the Slutsky equation as the compensating money income that will keep consumers at their same level of real income in the face of changing consumer price and factor prices.[30] The second part reflects the influence of initial distorting taxes upon the equilibrating change in lump sum taxes.

As we have discussed previously, if all initial commodity and factor taxes were zero or nondistorting, the second term of equation (41) drops out and the change in lump sum taxes required to equilibrate the government budget constraint exactly equals the payments required to keep consumers at their same level of welfare. Hence, as long as we ignore second order effects, there is no deadweight loss involved in raising revenue through distorting commodity and factor taxes and returning it as a lump sum to consumers.

When initial taxes are distorting, however, eq. (41) indicates that the change in lump sum taxes that equilibrates the government budget constraint is less than the change in lump sum taxes required to return consumers to their initial level of welfare. Hence a deadweight loss is involved in the introduction of distorting taxes in the presence of initial distoring taxes that does not occur in their absence. This deadweight loss equals the second term of eq. (41), which is equal to the change in government revenues arising from the substitution effects in consumption production, corrected by the factor $\mu = 1/m$. As pointed out above, m equals $1 - \sum_j \zeta_j \eta_j$. This measures the net effect upon revenue of a change in taxes.

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[30] In consumer theory, the compensating income in the face of changing consumer prices is $\sum_j C_j dq_j$. Part of that compensation is, however, already taken into account in the context of this model by the change in factor income due to changes in r and w.

6. Differential Incidence Effects

Equations (31) and (40) respectively express the differentials of the market clearing equations and the government budget constraint in terms of relative factor prices $(\hat{r}-\hat{w})$, the commodity tax coefficients $(\hat{T}_j)$, the factor tax coefficients $(\hat{T}_{ij})$, and the real lump sum tax $(\hat{H}-\hat{w})$.[31] Hence we have two equations in eight variables. Of these variables, $(\hat{r}-\hat{w})$ will always be treated as endogenous, and the government will be interpreted as having six degrees of freedom among the seven tax instruments.

6.1 The General Equation of Marginal Differential Incidence

By eliminating $(\hat{r}-\hat{w})$ from equations (31) and (40), we can obtain the following linear equation expressing the six degrees of freedom among the seven tax instruments, which we refer to as the general equation of marginal differential incidence

$$\sum_j m_j\hat{T}_j + \sum_{i,j} m_{ij}\hat{T}_{ij} + mh(\hat{H}-\hat{w}) = 0 \,. \tag{42}$$

The coefficients of this equation are given as follows.

$$m_h = mh(\hat{H}-\hat{w}) \tag{42.1}$$

$$m_1 = m\gamma_1 + E + \frac{\alpha}{\beta}(\tilde{\sigma}_D + \tilde{\eta}\,\gamma_1) \tag{42.2}$$

$$m_2 = m\gamma_2 - E - \frac{\alpha}{\beta}(\tilde{\sigma}_D - \tilde{\eta}\,\gamma_2) \tag{42.3}$$

$$m_{K1} = \theta_{K1}m_1 + \epsilon_1 + \frac{\alpha}{\beta}s_1 \tag{42.4}$$

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[31] Specifying changes in the tax structure in terms of percentage changes in the tax coefficients is analytically most convenient, given the way the model is developed. This requires a certain amount of interpretation though. Note that $d\tau_j = (1+\tau_j)\hat{T}_j$ and $d\tau_{ij} = (1+\tau_{ij})\hat{T}_{ij}$. Thus the $\hat{T}_j$ (or $\hat{T}_{ij}$) only correspond to changes in the ad valorem rates if there are no initial taxes.

$$m_{L1} = \theta_{L1}m_1 - \epsilon_1 - \frac{\alpha}{\beta} s_1 \qquad (42.5)$$

$$m_{K2} = \theta_{K2}m_2 + \epsilon_2 + \frac{\alpha}{\beta} s_2 \qquad (42.6)$$

$$m_{L2} = \theta_{L2}m_2 - \epsilon_2 - \frac{\alpha}{\beta} s_2 \qquad (42.7)$$

where $\alpha \equiv m\varphi - (E|\theta| + \epsilon)$

$$\beta \equiv |\theta|(\sigma_S + \tilde{\sigma}_D) - \tilde{\eta}\varphi \, .$$

Each of the terms defined by eqs. (42.1)-(42.7) represents the impact (as a fraction of disposal income y) on government revenue of a one percent change in the corresponding tax coefficient, after all the general equilibrium adjustments have been taken into account. The economic content of each coefficient is readily explained. Equation (42.1) indicates that a one percent change in real lump sum taxes changes government revenue by mh percent of disposable income. If both goods are not inferior (both η_j are positive) and there is initial taxation, m is less than one. Consequently the income-effect repercussions of the initial tax structure permit the government ultimately to retain less than 100 percent of every dollar levied by lump sum taxes.

Equations (42.2)-(42.7) describe the general equilibrium effect of a one percent change in the six commodity or factor tax coefficients. Each of these consists of three components. The first term describes the immediate impact on commodity tax collections caused by a direct change in the commodity tax rates or the price changes due to factor tax changes. Because this first term represents a real income loss to the consumer, it has an effect similar to that of a change in real lump sum taxes. Hence, these coefficients are all multiplied by the factor m. The second term describes the deadweight loss effect on government revenue caused by the substitution effects in consumption or production caused by initial distorting taxes. As indicated previously, in the absence of distorting initial taxes, these terms would all equal zero. The third term represents the effect on government tax collections acting through the change in the rental/wage ratio $(\hat{r}-\hat{w})$ necessary to satisfy the market clearing equation (31). Note that α and β are the coefficients of $(\hat{r}-\hat{w})$ respectively of the government budget constraint (40) and the market clearing equation (31).

6.2 Incidence Effects with $(\hat{H}-\hat{w})$ Endogenous

The expressions m_j, m_{ij}, and m_h, given in eqs. (42.1)-(42.7), give all the information relevant to the government in deciding to change the tax structure while keeping real government demand constant and the budget in equilibrium. The ratio of any two of the expressions (42.1)-(42.7) represents the marginal rate at which percentage changes in any two tax coefficients can be substituted for each other.[32] Furthermore, substitution of these compensating tax changes into the differential of the market clearing equation (31) determines the effect on the rental-wage ratio.

It has been customary in the literature to discuss the case where one (or more) factor or commodity taxes are changed, while the lump sum tax is adjusted to keep the private and government sectors in equilibrium. Often, however, the mechanics of the lump sum tax adjustments are kept in the background (by choosing the government budget as the equation to be eliminated by Walras' law or by setting initial taxes equal to zero). With the simplifying assumption of no initial distorting taxes, such a procedure loses nothing. With initial distorting taxes, however, it is useful to make the lump sum tax adjustment explicit, since it highlights the deadweight loss effects of these taxes. We therefore review this case within our general equilibrium framework and compare the resulting expression for changes in relative factor prices with earlier ones.

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[32] Inspection of eqs. (42.1)-(42.7) indicates that if initial taxes are zero or nondistorting, the equal yield conditions imposed by Mieszkowski (1967) will be correct. If initial taxes are distorting, however, the full expression for each m_j and m_{ij} must be considered.

We thus solve for $(\hat{H}-\hat{w})$ in equation (42), substitute the resulting expression into the market clearing equation (31), and solve for $(\hat{r}-\hat{w})$ to obtain

$$(\hat{r}-\hat{w}) = \frac{[(\hat{T}_1-\hat{T}_2)+(\hat{T}_{\ell 1}-\hat{T}_{\ell 2})][\tilde{\eta}\mu E-\tilde{\sigma}_D] + \sum_j(\hat{T}_{Kj}-\hat{T}_{Lj})(\tilde{\eta}\mu\epsilon_j - s_j)}{|\theta|(\sigma_S+\sigma_D) - \tilde{\eta}\mu(E|\theta|+\epsilon)} \qquad (43)$$

The denominator of eq. (43) represents the induced effects upon $(\hat{r}-\hat{w})$ due to a change in the tax structure. The first term represents the substitution effects in production and consumption due to a change in the tax structure, while the last term measures the income effect due to the initial distorting tax structure. The numerator measures the sum of the direct effects upon $(\hat{r}-\hat{w})$ due to a change in the tax structure. The term in the first bracket of the numerator measures the demand effects of the direct change in the commodity tax coefficients caused by the increase in commodity and factor taxes: it consists of substitution effects ($-\sigma_D$) and income effects arising from existing distorting taxes ($\tilde{\eta}\mu E$). The second bracket measures the factor substitution effects arising from increases in the distorting sectoral factor taxes, ($\hat{T}_{Kj} - \hat{T}_{Lj}$). Note that these factor substitution effects have an impact both through the supply side ($-s_j$) and the demand side ($\tilde{\eta}\mu\epsilon_j$), which comes from the deadweight loss in factor tax revenue, arising from initial factor taxes.

By examining the numerator of equation (43) we can obtain several tax equivalences immediately. First, equal proportional change in all taxes ($\hat{T}_j = \hat{T}_{ij}$, $j = 1,2$, $i = K,L$), equal proportional changes in commodity taxes ($\hat{T}_1 = \hat{T}_2$, $\hat{T}_{ij} = 0$), and equal proportional changes in factor taxes ($\hat{T}_{Kj} = \hat{T}_{Lj}$, $\hat{T}_j = 0$, $j = 1,2$) are all neutral and leave the rental-wage ratio unchanged. Thus general commodity taxes are equivalent to general factor taxes and both taxes are neutral in this world of inelastically supplied factors. Second, equal proportional changes in factor taxes imposed in sector j will have the same affect on the rental-wage ratio as an equal proportional change in the commodity tax on good j ($\hat{T}_{Kj} = \hat{T}_{Lj} = \hat{T}_j$). Thus general factor taxes imposed on sector j are equivalent to a commodity tax imposed on sector j. Since, however, all taxes are not being changed equally, factor taxes or commodity taxes in sector j will not be neutral. Note, however, that the neutrality and tax equivalences implied in equation (43) depend upon the neutrality of the endogenous tax change, ($\hat{H} - \hat{w}$).

It is instructive to compare the expression for the change in relative factor prices given in equation (43) with that given by Harberger (1962) and Mieszkowski (1967), whom we refer to as HM. HM consider a tax on capital in sector 1, compensated by a change in the lump sum tax. In the context of our analysis, their equation reads[33]

$$(\hat{r}-\hat{w}) = \frac{\hat{T}_{K1}(-\theta_{K1}\sigma_D - s_1)}{|\theta|(\sigma_S+\sigma_D)} \tag{44}$$

while equation (43) reduces to

$$(\hat{r}-\hat{w}) = \frac{\hat{T}_{K1}[-\theta_{K1}\tilde{\sigma}_D - s_1 + \tilde{\eta}\mu(E\theta_{K1}+\epsilon_1)]}{|\theta|(\sigma_S+\tilde{\sigma}_D) - \tilde{\eta}\mu(E|\theta| + \epsilon)} \tag{45}$$

The differences between eqs. (44) and (45) arise primarily from the existence of initial distorting taxes. First, eq. (45) contains the deadweight terms in the numerator and denominator. As we have discussed previously, in the absence of initial distorting taxes, these terms would drop out. Second, the demand elasticity of

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[33] Remember that $s_1 = S_{11} + S_{21}$ and $S_1 = S_{11} + S_{12}$. By expanding S_{11}, S_{12}, and S_1 we obtain

$$S_{11} = \frac{\lambda_{K2}\lambda_{L2}}{|\lambda|}\left[\frac{K_1}{K_2}\theta_{L1} + \frac{L_1}{L_2}\theta_{K1}\right]\sigma_1$$

$$S_{12} = \frac{\lambda_{K2}\lambda_{L2}}{|\lambda|}\sigma_2$$

$$S_1 = \frac{\lambda_{K2}\lambda_{L2}}{|\lambda|}\left[\left(\frac{K_1}{K_2}\theta_{L1} + \frac{L_1}{L_2}\theta_{K1}\right)\sigma_1 + \sigma_2\right]$$

where $\lambda_{K1}\lambda_{L2}/|\lambda| = 1/(K_1/K_2 - L_1/L_2)$. In our notation, $f_K = \theta_{K1}$, $f_L = \theta_{L1}$, $g_K = \theta_{K2}$, $S_x = -\sigma_1$, and $S_y = -\sigma_2$. Remembering that $|\theta| = \theta_{K1} - \theta_{K2}$, the equivalence with Harberger and Mieszkowski follows immediately.

substitution σ_D has been replaced by a weighted demand elasticity of substitution $\tilde{\sigma}_D$. Note that $\sigma_D \equiv -(E_{11} + E_{22})$ while $\tilde{\sigma}_D \equiv -(g_1E_{11} + g_2E_{22})$. Thus if real government demand were zero $C_j = X_j$, $g_j = 1$, and $\sigma_D = \tilde{\sigma}_D$. Third, in the presence of distorting factor taxes, the supply elasticity σ_S take place along a distorted transformation curve. Consequently in the presence of distorting taxes, for any given relative price ratio, the numerical value of σ_S given in eq. (44) will differ from that in eq. (45); the former measures the supply elasticity along a fixed transformation curve, while the latter measures the supply elasticity along a shrunken transformation curve. In the absence of any initial taxes, eqs. (44) and (45) are identical. In the absence of initial distorting taxes, equation (45) differs from equation (44) only in that the former contains $\tilde{\sigma}_D$ while the latter contains σ_D.

Initial distorting factor and commodity taxes create difficulties in predicting the differential incidence of a given tax change, for they imply that we must not only consider the relevant supply and demand elasticities and factor intensities, but also the deadweight effects. In the absence of distorting taxes Harberger (1962) has shown that it is possible to reach a number of conclusions about the presumptive sign of $(\hat{r}-\hat{w})$ when the tax on capital in sector 1 is altered. Examination of eq. (45) indicates, however, that in the presence of distorting initial taxes such conclusions are not generally justified. Depending on the nature of the existing taxation, the signs of E and ϵ_1 can be positive or negative. Moreover, as Jones (1971) has pointed out, in the presence of distorting taxes the sign of $|\theta|$ need not reflect differential capital intensities.[34] Thus the analysis of differential incidence generally requires knowledge of the existing tax structure.

6.3 Incidence Effects with $(\hat{H} - \hat{w}) = 0$

Since the government typically does not have lump sum taxes at its disposal, it is interesting to analyze the incidence effects of tax changes when the government must adjust factor or commodity taxes to maintain equilibrium. Examination of equations (42) and (31) indicate that equal endogenous adjustments in commodity taxes $(\hat{T}_1 = \hat{T}_2 = \hat{T}_c)$ or in factor taxes $(\hat{T}_{Kj} = \hat{T}_{Lj} = \hat{T}_\ell)$ affect the system in an identical fashion to adjustments in real lump sum taxes. Assume, for example, that the government changes factor tax coefficients while adjusting $\hat{T}_c$ to maintain equilibrium. It is straightforward to show that

$$(\hat{r} - \hat{w}) = \frac{(\hat{T}_{\ell 1} - \hat{T}_{\ell 2})(\tilde{\eta}\mu E - \tilde{\sigma}_D) + \sum_j (\hat{T}_{Kj} - \hat{T}_{Lj})(\tilde{\eta}\mu \epsilon_j - s_j)}{|\theta|(\sigma_S + \tilde{\sigma}_D) + \tilde{\eta}\mu(E|\theta| + \epsilon)} . \tag{46}$$

Similarly, if the government changes commodity tax coefficients while adjusting $\hat{T}_\ell$ to maintain equilibrium

$$\hat{r} - \hat{w} = \frac{(\hat{T}_1 - \hat{T}_2)(\tilde{\eta}\mu E - \tilde{\sigma}_D)}{|\theta|(\sigma_S + \tilde{\sigma}_D) - \tilde{\eta}\mu(E|\theta| + \epsilon)} . \tag{47}$$

Thus if the government adjusts general sales taxes or general income taxes to maintain equilibrium, the incidence effects of other taxes are identical to the incidence effects of these same taxes if the government adjusted real lump sum taxes. This conclusion should not be surprising, since changes in real lump sum taxes, income taxes, and sales taxes are all neutral and leave relative prices unchanged in this world of inelastically supplied factors.

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[34] $|\theta| = \theta_{K1} - \theta_{K2}$. But $\theta_{Kj} = r_j K_j / p_j X_j = \varphi_{Kj} + \zeta_{Kj}$ where $\varphi_{Kj} = rK_j / p_j X_j$ and $\zeta_{Kj} = \tau_{Kj} rK_j / p_j X_j$. Therefore $|\theta| = (\varphi_{K1} - \varphi_{K2}) + (\zeta_{K1} - \zeta_{K2})$. The term $\varphi_{K1} - \varphi_{K2}$ reflects the relative capital intensities in the two sectors and will be positive if sector 1 is more capital intensive. But if capital in sector 2 is taxed more heavily than that in sector 1, $|\theta|$ may be negative.

Of course, constraining the government to maintain equilibrium by adjusting sales or income taxes removes several degrees of freedom. Consequently, let us alternatively consider the case where the government adjusts $\hat{T}_2$ to maintain equilibrium in response to exogenous changes in other factor or commodity tax coefficients. Thus we solve for $\hat{T}_2$ in eq. (42) and substitute the resulting expression into eq. (31) to obtain

$$(\hat{r}-\hat{w}) = \frac{(\hat{T}_1 + \hat{T}_{\ell 1})(\tilde{\eta}\mu E - \tilde{\sigma}_D) + \sum_j (\hat{T}_{Kj} - \hat{T}_{Lj})[\gamma_2(\tilde{\eta}\mu\epsilon_j - s_j) - \mu(\epsilon_j - s_j E)]}{|\theta|(\sigma_S + \tilde{\sigma}_D)(\gamma_2 - \mu E) + \mu(E|\theta| + \epsilon)(\tilde{\sigma}_D - \tilde{\eta}\gamma_2) + \varphi(\tilde{\eta}\mu E - \tilde{\sigma}_D)} \; . \qquad (48)$$

In this case, equal changes in factor tax coefficients (i.e., $\hat{T}_{Kj} = \hat{T}_{Lj}$ and $\hat{T}_1 = 0$) will not be neutral. This is not surprising since the equilibrating change in $\hat{T}_2$ will be different from zero. Because $\hat{T}_2 \neq \hat{T}_1$, relative prices will change, even though the exogenous tax change was ostensibly neutral.

A comparison of eq. (48) with eq. (43) indicates that the primary difference in the expression for $(\hat{r}-\hat{w})$ lies in the γ_2 factors and the increased number of deadweight loss effects in eq. (48). The γ_2 factor represents the share of disposable income spent on C_2, and therefore reflects a weighting factor associated with the endogenous tax term. The increased number of deadweight loss terms reflects the distorting nature of the endogenous tax adjustment.

The difference between the factor price adjustments when neutral and non-neutral taxes are used to maintain equilibrium in the government sector can be seen most clearly if we consider a change in the tax coefficient on capital in sector 1, compensated respectively by $(\hat{H} - \hat{w})$ and $\hat{T}_2$. In the first case, we obtain eq. (45). In the second case we obtain

$$(\hat{r}-\hat{w}) = \frac{\hat{T}_{K1}[\theta_{K1}(\tilde{\eta}\mu E - \tilde{\sigma}_D) + \gamma_2(\tilde{\eta}\mu\epsilon_1 - s_1) - \mu(\epsilon_1\tilde{\sigma}_D - s_1 E)]}{|\theta|(\sigma_S + \tilde{\sigma}_D)(\gamma_2 - \mu E) + \mu(E|\theta| + \epsilon)(\tilde{\sigma}_D - \tilde{\eta}\gamma_2) + \varphi(\tilde{\eta}\mu E - \tilde{\sigma}_D)} . \qquad (49)$$

The first two terms of the numerator in eq. (49) appear in eq. (45), with the exception of the γ_2 factor on the second term. The third term of the numerator does not appear in eq. (45), however; it reflects a further deadweight loss effect due to the distorting nature of the endogenous tax adjustment. Similarly, the denominator of eq. (49) contains the denominator of eq. (45), weighted by the factor γ_2, plus additional deadweight loss effects.[35]

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[35] We can rewrite the denominator of eq. (49) as

$$\gamma_2[\,|\theta|(\sigma_S + \tilde{\sigma}_D) - \tilde{\eta}\mu(E|\theta| + \epsilon)] + \mu\tilde{\sigma}_D(E|\theta| + \epsilon) + \varphi(\tilde{\eta}\mu E - \tilde{\sigma}_D)$$

The first term of this expression is the denominator of eq. (45), multipied by γ_2. The last two terms of this expression reflect further deadweight effects.

7. Summary and Conclusions

Differential tax incidence has previously been analyzed in a general equilibrium framework that assumes initial nondistorting taxes. In this case, the general equilibrium impacts of a change in the tax structure can be described by substitution effects in consumption and production alone. Moreover, in such a world, there is no deadweight loss associated with the introduction of marginal distorting taxes. Thus the general equilibrium impacts of changes in the tax structure are qualitatively similar to the partial equilibrium effects of such a change.

This paper has stressed, however, that the existence of initial distorting taxes changes the general equilibrium response of the system to changes in the tax structure in a qualitative way, by making it necessary to consider explicitly income effects in consumption and revenue effects in the government budget constraint. When the initial tax structure is non-neutral, moreover, distorting changes in the tax structure will create a deadweight loss effects. Thus to analyze the incidence effects of changes in the tax structure we not only need to know the substitution effects in consumption and production, but also the income and deadweight loss effects.

Of course, whether initial taxes are distorting or not is an empirical question. But the present analysis indicates that the existing structure of government taxation can have a profound effect upon the general equilibrium response of the system to a given change in the tax structure and should not, therefore, be ignored.

REFERENCES

Feldstein, Martin S., "Tax Incidence in a Growing Economy with Variable Factor Supply," (1972; mimeo).

Friedlaender, Ann F. and Due, John F., "Tax Burden, Excess Burden and Differential Incidence Revisited," Public Finance, February 1973.

Harberger, A.C., "The Incidence of the Corporate Income Tax," Journal of Political Economy, June 1962.

Hatta, T., "Reducing Price Distortion: How to Avoid the 'Second Best' Problem in Policy Formations," (1972; mimeo).

Jones, R.W., "The Structure of Simple General Equilibrium Models," Journal of Political Economy, December 1965.

_____, "Distortion in Factor Markets and the General Equilibrium Model of Production," Journal of Political Economy, May/June 1971.

McLure, Charles, "Tax Incidence, Macroeconomic Policy, and Absolute Prices," Quarterly Journal of Economics, May 1970.

Mieszkowski, P.M., "On the Theory of Tax Incidence," Journal of Political Economy, June 1967.

_____, "Tax Incidence Theory: The Effects of Taxes on the Distribution of Income," Journal of Economic Literature, December 1969.

Musgrave, R.A. "On Incidence," Journal of Political Economy, August 1953.

_____, The Theory of Public Finance (New York: McGraw Hill, 1959).

Samuelson, Paul A., "Social Indifference Curves," Quarterly Journal of Economics, Vol. LXX, No. 1 (February, 1956), pp. 1-28.

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